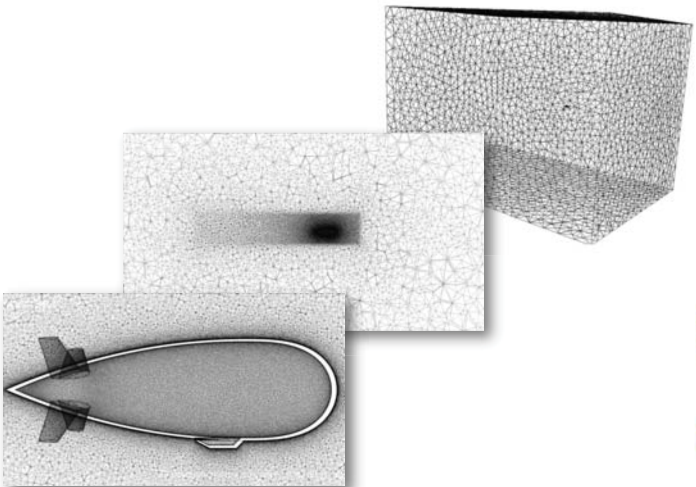
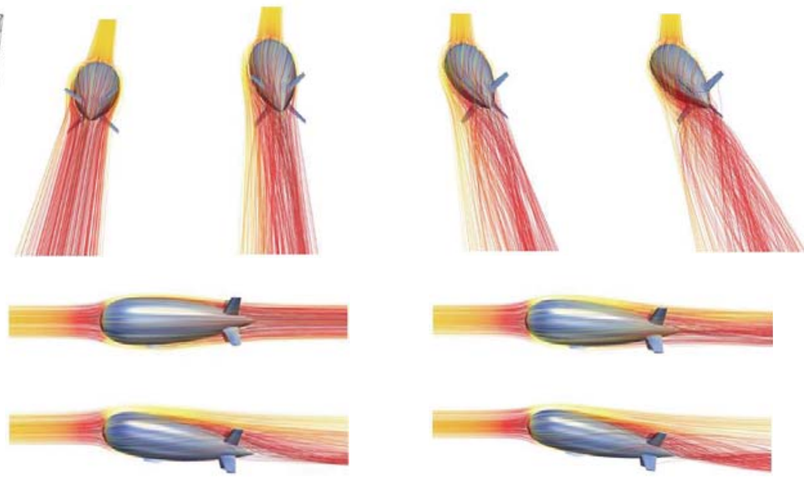
Anisotropic adaptive stabilized finite element solver for RANS models

STRATOBUS™

Ghalia GUIZA Alexandre BOILLEY Philippe MELIGA Elie HACHEM
Computing & Fluids Research Group
MINES ParisTech, PSL - Research University, CEMEF, 06904 Sophia-Antipolis

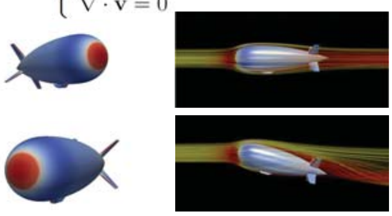




Computational Fluid Dynamics (CFD)

- Cimlib-CFD : C++ library developed at CEMEF
- Incompressible Navier-Stokes equations:

$$\begin{cases} \rho\left(\partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v}\right) - \nabla\cdot\boldsymbol{\sigma} = \mathbf{F} \\ \nabla\cdot\mathbf{v} = 0 \end{cases}$$



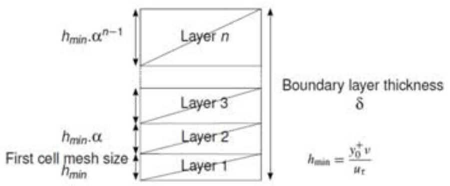
- Variational MultiScale (VMS) stabilized finite element method[1]

$$\begin{cases} \rho\left(\partial_t \mathbf{v}_h, \mathbf{w}_h\right)_\Omega + \left(\rho\mathbf{v}_h\cdot\nabla\mathbf{v}_h, \mathbf{w}_h\right)_\Omega \\ \qquad - \sum_{K\in\mathcal{T}_h}\left(\tau_1\mathcal{R}_{\mathrm{M}}, \rho\mathbf{v}_h\nabla\mathbf{w}_h\right)_K + \left(2\mu\varepsilon(\mathbf{v}_h):\varepsilon(\mathbf{w}_h)\right)_\Omega \\ \qquad - \left(p_h, \nabla\cdot\mathbf{w}_h\right)_\Omega + \sum_{K\in\mathcal{T}_h}\left(\tau_2\mathcal{R}_{\mathrm{C}}, \nabla\cdot\mathbf{w}_h\right)_K = \left(\mathbf{f}, \mathbf{w}_h\right)_\Omega \\ \left(\nabla\cdot\mathbf{v}_h, q_h\right)_\Omega - \sum_{K\in\mathcal{T}_h}\left(\tau_1\mathcal{R}_{\mathrm{M}}, \nabla q_h\right)_K = 0, \quad \forall q_h\in Q_h \end{cases}$$

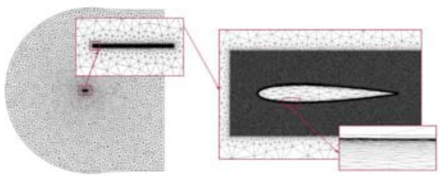
[1] *E. Hachem, B. Rivaux, T. Kloczko, H. Digonnet, and T. Coupez Stabilized finite element method for incompressible flows with high Reynolds number*

Anisotropic Boundary layer Mesh

- Levelset Framework to capture/localize the boundary layer[2]



- Parallel mesher and remesher

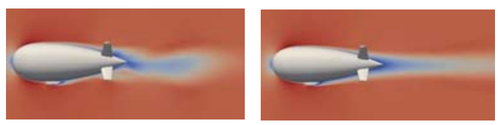


[2] *L. Billon, Y. Mesri, and E. Hachem, Anisotropic boundary layer mesh generation for immersed complex geometries*

Turbulence modeling

- Spalart-Allmaras (SA) turbulence model solved using Streamline Upwind Petrov-Galerkin (SUPG)[3]

$$\frac{\tilde{\nu}^{n+1}-\tilde{\nu}^{n}}{\Delta t} + \underbrace{\left(\mathbf{v}^{n+1} - \frac{c_{b2}}{\sigma}\nabla\tilde{\nu}^{n+1}\right)\cdot\nabla\tilde{\nu}^{n+1}}_{\text{convection}} - \underbrace{\frac{1}{\sigma}\nabla\cdot\left[(\nu+\tilde{\nu}^{n+1})\nabla\tilde{\nu}^{n+1}\right]}_{\text{diffusion}}$$
$$- \underbrace{\left[c_{b1}(1-f_{t2}^{n+1})\tilde{S}^{n+1} + \left(c_{w1}f_w^{n+1} - \frac{c_{b1}}{\kappa^2}f_{t2}^{n+1}\right)\frac{\tilde{\nu}^{n+1}}{d^2}\right]\tilde{\nu}^{n+1}}_{\text{reaction}} =$$



$$\left(\frac{\tilde{\nu}^{n+1,i+1}-\tilde{\nu}^{n}}{\Delta t}, \omega_h\right)_\Omega +$$
$$\left(\left[\mathbf{v}^{n+1} - \frac{c_{b2}}{\sigma}\nabla\tilde{\nu}^{n+1,i}\right]\cdot\nabla\tilde{\nu}_h^{n+1,i+1}, \omega_h\right)_\Omega - \left(\frac{1}{\sigma}(\nu+\tilde{\nu}_h^i)\nabla\tilde{\nu}_h^{n+1,i+1}, \nabla\omega_h\right)_\Omega$$
$$- \left(\left[c_{b1}(1-f_{t2}^{n+1})\tilde{S}_h^{n+1} + \left(c_{w1}f_w^{n+1} - \frac{c_{b1}}{\kappa^2}f_{t2}^{n+1}\right)\frac{\tilde{\nu}_h^{n+1,i}}{d^2}\right]\tilde{\nu}_h^{n+1,i+1}, \omega_h\right)_\Omega$$
$$+ \sum_K\left(\mathcal{R}(\tilde{\nu}^{n+1,i}), \tau_3^{n+1,i}\left[\mathbf{v}_h^{n+1} - \frac{c_{b2}}{\sigma}\nabla\tilde{\nu}_h^{n+1,i}\right]\cdot\nabla\omega_h\right)_K = 0, \quad \forall\omega_h\in W_h$$

[3] *J. Sari, F. Cremonesi, M. Khalloufi, F. Cauneau, Y. Mesri, and E. Hachem, Anisotropic adaptative stabilized finite element solver for RANS models*